Authenticating Aggregate Queries over Set-Valued Data with Confidentiality

Cheng Xu, Qian Chen, Haibo Hu, Jianliang Xu, Xiaojun Hei

Abstract—With recent advances in data-as-a-service (DaaS) and cloud computing, aggregate query services over set-valued data are becoming widely available for business intelligence that drives decision making. However, as the service provider is often a third-party delegate of the data owner, the integrity of the query results cannot be guaranteed and is thus imperative to be authenticated. Unfortunately, existing query authentication techniques either do not work for set-valued data or they lack data confidentiality. In this paper, we propose authenticated aggregate queries over set-valued data that not only ensure the integrity of query results but also preserve the confidentiality of source data. As many aggregate queries are composed of multiset operations such as set union and subset, we first develop a family of privacy-preserving authentication protocols for primitive multiset operations. Using these protocols as building blocks, we present a privacy-preserving authentication framework for various aggregate queries and further optimize their authentication performance. Security analysis and empirical evaluation show that our proposed privacy-preserving authentication techniques are feasible and robust under a wide range of system workloads.

Index Terms—Query Authentication, Aggregate Queries, Set-Valued Data, Merkle Hash Tree

1 Introduction

Recent advances in data-as-a-service (DaaS) and cloud computing have been driving data from different sources into massive repositories for queries. A typical example is BigQuery [1] in the Google Cloud Platform, which uses SQL-like queries to analyze big datasets. The results of these queries, particularly aggregate queries, in turn serve as sources for data analytics. For example, business intelligence executives can thus make critical, million-dollar decisions such as investing in new business opportunities. As such, ensuring the integrity of query results from the service provider is crucial. Similar requirements also exist in many other domains such as scientific research and government policy. For example, personal genomics analysis (e.g., 23andMe and the Personal Genome Project (PGP) at Harvard Medical School [2]) is based on aggregate queries on large genome datasets, the integrity of whose results is vital. Below is one example:

PID	ZIP	Mut-Genes
P1	95014	A-C130R, P-I696M
P2	20482	H-C282Y, P-P12A, R-G1886S
P3	95014	A-C130R, U-G71R, W-R611H
P4	01720	A-V2050L, H-C282Y, M-R52C, U-G71R
P5	20134	A-C130R, P-P12A, R-G1886S, S-E366K
P6	17868	C-R102G, R-G1886S
P7	55410	C-R102G, C-Q1334H, S-E288V
P8	20852	C-R102G, P-P12A, R-G1886S, K-T220M

TABLE 1: Set-Valued Genome Dataset

Example 1. **Aggregate Queries on PGP Data.** Table 1 shows a sample genome dataset, where *PID* is participant ID, *ZIP* is ZIP code, and *Mut-Genes* is a sensitive set-valued attribute that records the mutation genes of each participant. Users (e.g., medical doctors) may be interested in the following aggregate queries:

- ***Q1***: *Find the most common gene in the district of Cupertino, CA (ZIP: 95014).*
- ***Q2***: *Count the number of participants who carry the gene 'R-G1886S'.*
- ***Q3***: *Find the most frequent genes with supports $\geq$ 3 in ZIPs 20***.*

The corresponding query results are: {'A-C130R'}, 4, and {'P-P12A', 'R-G1886S'}, respectively.

Unfortunately, there is no guarantee of the service provider returning correct and complete results, for various reasons that could include service outages, hack attacks, or even corporate dishonesty. In the field of query processing, authentication on query results has been studied by a large body of literature [3]–[10]. The fundamental concept is that the data owner signs a well-designed *authenticated data structure* (ADS), based on which the service provider then constructs a cryptographic proof for each query and returns it along with the query results for the query client to verify. However, the prior research cannot be applied to aggregate queries on set-valued genomics or business data as exemplified above for two reasons. First, most of the research has been focused on relational or spatial data and their associated query types, whereas set-valued data have their own unique operations and aggregate query types. Second, most of the research ignores data confidentiality — they assume that the service provider is willing to and capable of sharing a substantial amount of source data to the query client for result verification. In the genomics analysis, business intelligence, and many other cases, however, the source data cannot be disclosed to the query client, either because they are private assets of the data owner, or

- *Cheng Xu, Qian Chen and Jianliang Xu are with the Department of Computer Science, Hong Kong Baptist University, Hong Kong. E-mail: {chengxu, qchen, xujl}@comp.hkbu.edu.hk*
- *Haibo Hu is with the Department of Electronic and Information Engineering, Hong Kong Polytechnic University, Hong Kong. E-mail: haibo.hu@polyu.edu.hk*
- *Xiaojun Hei is with the School of Electronic Information and Communications, Huazhong University of Science and Technology, Wuhan, China. E-mail: heixj@hust.edu.cn*

because they may contain sensitive personal information such as individual genome sequences.

As set-valued data are ubiquitous and sometimes indispensable in describing real-world problems for analytics, in this paper we study authenticated aggregate query services over set-valued data with confidentiality preservation. We assume that a dataset consists of one sensitive set-valued attribute (e.g., mutation-gene set) and multiple non-sensitive attributes (e.g., ZIP code and age). As illustrated in Example 1, an aggregate query is defined as a query whose result is derived from aggregates of data. In this paper, each aggregate query consists of two phases: a filtering phase that filters data by a range selection on non-sensitive attributes, followed by an aggregating phase that aggregates on the sensitive set-valued attribute. For broader applicability, we model the sensitive set-valued attribute as a *multiset*, i.e., a set that allows duplicate elements. This enables advanced data analytics such as frequent itemset mining in market basket analysis [11], inverted indexes in web search and auto-completion [12], and web log mining [13], all of which require a collection data type that supports duplicate elements.

Because aggregate queries exhibit unique properties that are unseen in those queries previously studied for authentication [3]–[5], the design of the ADS and its related authentication techniques face great challenges. First, due to the dynamics of aggregate queries (e.g., each query in Example 1 selects a different group of participants), it is impossible to pre-compute the query results and endorse them in advance. Second, aggregate queries over set-valued data are largely supported by multiset operations, for which existing authentication techniques, mostly designed for relational queries, cannot be applied. Third, to safeguard source data confidentiality, the query client must be able to verify the query results without learning any single set of sensitive values. To address these challenges, we propose PA^2, a privacy-preserving authentication framework for aggregate queries. It consists of an authenticated index *Merkle Grid tree* (MG-tree), a family of privacy-preserving authentication protocols for primitive multiset operations, and a series of aggregate query processing algorithms. We further propose several optimizations to reduce the computation and communication overheads for both the server and the query client. In summary, our contributions made in this paper are as follows:

- To the best of our knowledge, this is the first work that addresses both integrity and confidentiality for aggregate queries over set-valued data.
- We propose a privacy-preserving authentication framework and we develop a set of privacy-preserving authentication protocols and algorithms for various aggregate queries.
- We provide formal security analysis and cost models for the proposed authentication protocols and algorithms.
- We propose optimizations to enhance the performance of query authentication algorithms.
- We perform extensive performance evaluation on real-world datasets. Empirical results demonstrate the feasibility and robustness of our proposals.

The rest of this paper is organized as follows. Section 2 reviews the related work on aggregate queries and authenticated query processing. Section 3 formally defines the problem and its security model. Section 4 introduces some related cryptographic constructs. Section 5 proposes the PA^2 framework for privacy-preserving authentication of aggregate queries based on a family of verifiable multiset operations. Security analysis and performance optimization techniques are given in Sections 6 and 7. Section 8 presents our experimental results and is followed by a conclusion.

2 Related Work

Aggregate Queries over Set-Valued Data. Set-valued data, in which a set of values is associated with an individual, is common in data analytics applications ranging from market basket analysis, to web log mining, to heathcare research. Özsoyoğlu *et al.* were the first to extend relational algebra and aggregate functions to set-valued data [14]. They defined algebraic expressions such as set union, set difference, and aggregation. Since then, data analytics over set-valued data has been extensively studied in various application domains. The most well-known application is association rule mining [15]. Given a database of sales transactions, each containing a subset of items in the product universe, the objective is to find rules such as "a user buying item(s) X will probably buy item(s) Y". Although various algorithms are proposed, a fundamental problem in this application is to efficiently compute two aggregate values, namely, the *support* and *confidence* of X [11]. The former is the number of transactions that contain X, whereas the latter is those in the former that also contain Y. With the boom of web search and online advertisement, query log and click stream have become new sources of set-valued data. A variety of aggregate queries have been proposed on these sources for tasks such as website clustering and frequent item identification and counting [16]. Recently, graphs have become another new source of set-valued data. In particular, social networks have contributed various social relations, such as "friend/unfriend," "follow," "post/tag," and user access rights, to set-valued data [17]. Aggregating such data for social network analysis and recommendation has been intensively studied [18].

Authenticated Query Processing. A large body of research on authenticated query processing has been carried out to verify the integrity of query results against an untrusted service provider. Most of the existing works focus on relational and spatial data [3]–[6], [19]. Their query authentication schemes are based on two basic techniques, namely, digital signature and Merkle hash tree (MHT). The former is a public-key message authentication scheme based on asymmetric cryptography. A digital signature is produced for each data value by the data owner using a private key. A verifier can verify the authenticity of a value using the owner's public key and the value's signature. To further ensure the completeness of query results, signature chaining was proposed to correlate the signatures of adjacent values [3]. Signature chaining is simple, but it requires each value to be signed and thus is not scalable to large datasets. MHT, on the other hand, requires only a single signature on an index tree [20]. Each entry in a leaf node is assigned a digest based on its data value, and each entry in an internal node is assigned a digest derived from its child nodes. The data owner signs the root digest of the MHT, which can be used to verify any subset of data values. The MHT has been widely adapted to various index structures. Typical examples include the Merkle B-tree for relational data [4], the Merkle R-tree for spatial data [5], [21], the authenticated inverted index for text data [22], and the authenticated prefix tree for multi-source data [9] .

More recently, privacy-preserving query authentication techniques have been studied for location-based range and top-k queries [23], [24]. The core idea is to enable the verification of comparison results on private values. However, these previous works consider relational/spatial data only, not set-valued data,

reference	set operations	multiset operations	aggregate queries	privacy preserving
[25], [26], [27]	✓	✗	✗	✗
[29]	✓	only *sum*	✗	✓
[30], [31]	✓	✗	✗	✓
our work	✓	✓	✓	✓

TABLE 2: Comparison with prior works

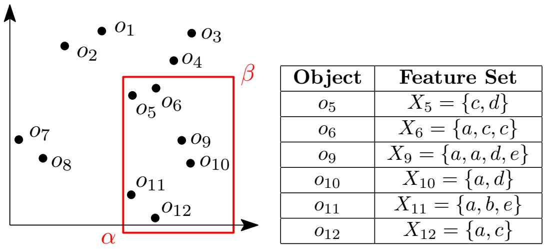
which are the focus of this paper. As we will see later in this paper, aggregate queries over set-valued data heavily involve primitive multiset operations such as set union and subset. This renders the existing privacy-preserving authentication techniques that are based on private value comparisons inapplicable.

On the other hand, there are only a few query authentication works over set-valued data. For primitive set operations, Papamanthou *et al.* proposed efficient verification protocols for set union, set intersection, and set difference operations [25]. Canetti *et al.* extended it by allowing data updates and monolithic verification of hierarchical set operations [26]. Papadopoulos *et al.* studied efficient high-dimensional range query authentication [27]. Dong *et al.* [28] developed a result verification protocol for frequent itemset mining. Yet, none of these previous studies has considered the data confidentiality requirement. For example, the solution in [28] allows the query client to learn which transactions contain the frequent itemsets, thus breaching data privacy. Protecting the confidentiality of source data requires designing new cryptographic constructs on primitive multiset operations. The preliminary work in [29] supports multiset sum operations only. Papadopoulos *et al.* proposed a zero-knowledge accumulator for set-valued data and common primitive operations [30]. However, it is computationally expensive and does not support multisets. Recently, another zero-knowledge set accumulator is proposed by Zhang *et al.* It is expressive but still lacks of the support for multiset operations [31]. Furthermore, how to support authenticated aggregate queries based on verifiable multiset operations while preserving data confidentiality is another new challenge. Table 2 summarizes the differences between our work and the previous works.

3 Problem Definition

A *data owner* (DO) owns a dataset $\mathbb{D} = \{o_1, o_2, \cdots, o_n\}$. Each object o_i is represented by $<A_i, X_i>$, where A_i is a set of non-sensitive attributes, and X_i is a sensitive multiset of *features* (hereafter called *feature set*). The DO outsources $\mathbb{D}$ to a third-party *service provider* (SP), together with an *authenticated data structure* (ADS) signed with the DO's private key. Based on this, the SP provides aggregate query services to clients (e.g., **Q1** and **Q2**, and **Q3** in Example 1).

Multiset Operations. A multiset is a generalization of a set in which elements are allowed to occur more than once [32]. The number of occurrences is called the *multiplicity* of an element. For instance, in multiset $\{a, a, b\}$, a has a multiplicity of 2 and b has a multiplicity of 1. The multiset is order insensitive, so it can be represented as a set of pairs (x, η) where x is an element and η is its multiplicity. The above multiset $\{a, a, b\}$ can thus be rewritten as $\{(a, 2), (b, 1)\}$. The most important operations in a multiset are *union* and *sum*, denoted as $\cup$ and $\uplus$, respectively. The *union* operation on multisets is exactly the same as that on regular sets — it simply unifies two multisets and sets the multiplicity of each element to 1. For instance, given multisets $X_1 = \{(a, 2), (b, 1)\}$, $X_2 = \{(b, 1), (c, 2)\}$, we have $X_1 \cup X_2 = \{(a, 1), (b, 1), (c, 1)\}$. In contrast, the *sum* operation sets the multiplicity of each element as the sum of multiplicities in the original multisets. For instance, given the same multisets as above, we have $X_1 \uplus X_2 = \{(a, 2), (b, 2), (c, 2)\}$.



Object	Feature Set
o_5	$X_5 = \{c, d\}$
o_6	$X_6 = \{a, c, c\}$
o_9	$X_9 = \{a, a, d, e\}$
o_{10}	$X_{10} = \{a, d\}$
o_{11}	$X_{11} = \{a, b, e\}$
o_{12}	$X_{12} = \{a, c\}$

(a) Objects (b) Features

Fig. 1: Example of Aggregate Queries

Aggregate Queries. Since the results of aggregate queries are derived from aggregates of data, our study mainly focuses on the following primitive aggregate queries: *max/min*, *count*, *sum*, *top-k*, and *frequent feature query (FFQ)*.[1] More specifically, an aggregate query can be expressed in the form of $Q = (q, \{x_i\}, [\alpha, \beta])$, where q is the aggregate operator, $\{x_i\}$ is the queried feature (which is only needed for *count* and *sum*), and $[\alpha, \beta]$ specifies the selection range on the non-sensitive attributes. Consider a sample query $Q = (q, \{x_i\}, [\alpha, \beta])$ in Figure 1. The query range $[\alpha, \beta]$ selects the objects $o_5, o_6, o_9, o_{10}, o_{11}, o_{12}$. Since $X_5 \uplus X_6 \uplus X_9 \uplus X_{10} \uplus X_{11} \uplus X_{12} = \{(a, 6), (b, 1), (c, 4), (d, 3), (e, 2)\}$, the query result is based on the aggregate operator q, as follows:

- $q = sum$ or $count$: This sums or counts the multiplicities of $\{x_i\}$ in all selected objects. Supposing $\{x_i\} = \{a\}$, the sum and $count$ results are $(a, 6)$ and $(a, 4)$, respectively.[2]
- $q = max$ or min: This selects the feature with the maximum or minimum summed multiplicity in all selected objects. If $q = max$, the result is $(a, 6)$; if $q = min$, the result is $(b, 1)$.
- $q =$ *top-k*: This selects the top-k features with the largest summed multiplicities in all selected objects. Supposing k = 3, the result is $(a, 6), (c, 4), (d, 3)$.
- $q =$ *FFQ$_\delta$*: This selects the features whose summed multiplicities in all selected objects are no less than a threshold δ. Supposing δ = 4, the result is $(a, 6), (c, 4)$.

The aggregate queries in Example 1 can be reduced to the above aggregate queries as follows. **Q1** is simply a max query, i.e., $(max,$ -, $[95014, 95014])$. **Q2** is a *count* query, i.e., $(count, \{R\text{-}G1886S\}, [00000, 99999])$. **Q3** is an FFQ query, $(FFQ_3, -, [20000, 29999])$, which finds the set of genes whose sum result is no less than 3.

Threat Model and Problem Statement. We consider two potential security threats: 1) the SP could provide unfaithful query execution, thereby returning incorrect or incomplete query results; and 2) data privacy could be breached if sensitive source data are disclosed to the query client. Thus, the authentication problem we are investigating is for the query client to verify that the SP executes Q faithfully in terms of the following conditions: 1) the candidate objects are correctly selected and no objects in the selection range are skipped; 2) the returned features and multiplicities are not tampered with; and 3) the query result satisfies the aggregation semantics. The confidentiality requirement in this problem is to protect the objects' (sensitive) feature sets against

[1] Extension to other more advanced aggregate queries such as *average* and *confidence* will be discussed in Section 5.2.3.

[2] The sum and $count$ results will be the same if there are no duplicate elements in the feature sets.

the query client. That is, the client cannot infer the features (as well as their multiplicities) of any single object beyond what is implied from the query result.

If neither efficiency nor confidentiality is a concern, authenticating an aggregate query can work as follows. The SP returns a *verification object* (VO) to the client, along with the query result. As a naive solution, the VO may include the non-sensitive attributes and sensitive features of all objects in $\mathbb{D}$ and a signature of $\mathbb{D}$. The client uses the VO to verify the soundness and completeness of the results by testing the following conditions:

- None of the objects in $\mathbb{D}$ is tampered with.
- All candidate objects are in $[\alpha, \beta]$ and no objects in $[\alpha, \beta]$ are missing.
- The features and multiplicities of the candidate objects are correct.
- The result satisfies the aggregation semantics of q.

However, the verification cost of this naive solution is prohibitively high because the entire dataset has to be returned. Moreover, verifying the last two conditions without disclosing sensitive feature sets requires privacy-preserving protocols. To address these issues, we propose an efficient privacy-preserving authentication framework based on verifiable multiset operations, the preliminaries of which are introduced in the next section.

4 Preliminaries

This section gives some preliminaries on cryptographic constructs and integrity assurance.

Cryptographic Hash Function. A cryptographic hash function $h(\cdot)$ accepts an arbitrary-length string as its input and returns a fixed-length bit string. It is collision resistant, i.e., it is difficult to find two different messages m_1 and m_2 such that $h(m_1) = h(m_2)$. Classic cryptographic hash functions include MD5 and SHA-1.

Bilinear-Map (BM) Accumulator. This maps a multiset to a single value for ease of processing. Let $\mathbb{G}$ be a cyclic multiplicative group of order p. A BM accumulator is a function of a multiset X of n elements in the cyclic group $\mathbb{Z}_p$ [33]. It returns an *accumulative value* of X:

$$acc(X) = g^{P(X)} = g^{\prod_{x \in X}(x+s)},$$

where g is a group generator of $\mathbb{G}$, $s \in \mathbb{Z}_p^* = \mathbb{Z}_p \backslash \{0\}$ is a random secret, and $P(X) = \prod_{x \in X}(x+s)$.

One useful property of $acc(X)$ is that even without knowing s, $acc(X)$ can still be computed by X and $g, g^s, \cdots, g^{s^k}$ $(k \geq |X|)$ through polynomial interpolation. As for its security, it has been proved in [34] that the accumulative function $acc(\cdot)$ is collision resistant.

Randomized BM Accumulator. The above accumulative value $acc(X)$ is deterministic for a fixed multiset X. As such, an adversary can determine with high confidence that two multisets are the same if they happen to have the same accumulative value. To enhance confidentiality, we propose randomizing the acc value of X as

$$acc(X) = g^{P(X) \cdot r_X}, \tag{1}$$

where r_X is a random value hidden from the query client but disclosed to the SP. It is worth noting that this randomization does not affect the original properties of a BM accumulator. We further prove in Section 6 that the randomized acc values are indistinguishable under chosen plaintext attack.

Bilinear Pairing. This maps a pair of elements in two groups to a single element in a third group. Let $\mathbb{G}_t$ be another cyclic multiplicative group with the same order p. We can find a bilinear mapping $e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_t$ which has the following properties:

1) *Bilinearity*: If $u, v \in \mathbb{G}$ and $e(u, v) \in \mathbb{G}_t$, then $e(u^a, v^b) = e(u, v)^{ab}$ for any u, v.
2) *Non-degeneracy*: $e(g, g) \neq 1$.
3) *Computability*: Given $u, v \in \mathbb{G}$, it is easy to compute $e(u, v)$.

Bilinear q-Strong Diffie-Hellman (DH) Assumption. This assumption shows that bilinear pairing is appropriate for multiset operation authentication as it is hard to forge. Let $(\mathbb{G}, \mathbb{G}_t, e, g)$ be a bilinear pairing. This assumption says that as long as $s \in \mathbb{Z}_p^*$ is secret, even given all elements $g, g^s, \cdots, g^{s^k} \in \mathbb{G}$, no probabilistic polynomial-time (PPT) algorithm can derive $e(g, g)^{1/(x+s)}$ for any $x \in \mathbb{Z}_p^*$ with a probability higher than a negligible value [35]. In essence, this bilinear q-strong DH assumption extends the regular DH assumption on g and $\mathbb{G}$ to $e(g, g)$ and $\mathbb{G}_t$. This assumption will be used as a foundation in our security analysis.

Set Operation Authentication. Based on the above, two result verification protocols have been introduced in [25], [26] to authenticate the following operations on two sets X_1 and X_2: (i) $X_1 \subseteq X_2$ and (ii) $X_1 \cap X_2 = \emptyset$.

For (i), the server computes a witness value $W = acc(X_2 - X_1)$ and returns it to the query client. The client then verifies $X_1 \subseteq X_2$ by checking:

$$e(acc(X_1), W) \stackrel{?}{=} e(acc(X_2), g).$$

For (ii), according to the extended Euclidean algorithm, there are two polynomials Q_1, Q_2 such that

$$Q_1 \cdot P(X_1) + Q_2 \cdot P(X_2) = 1.$$

As such, the server prepares $F_1 = g^{Q_1}, F_2 = g^{Q_2}$, and then the client verifies it by checking:

$$e(F_1, acc(X_1)) \cdot e(F_2, acc(X_2)) \stackrel{?}{=} e(g, g).$$

Though these two protocols are privacy-preserving in nature and can be extended to multisets, we have yet to design privacy-preserving authentication protocols for other multiset operations such as *sum* and *union*, which will be covered in Section 5.1.

5 PA^2: Privacy-Preserving Authentication Framework for Aggregate Queries

In this section, we present the complete framework that can authenticate various aggregate queries while preserving data confidentiality. Fig. 2 illustrates both the query and authentication flow charts of the framework. The framework consists of two phases: candidate object selection and aggregate query processing. Recall that given an aggregate query $Q = (q, \{x_i\}, [\alpha, \beta])$, the SP first selects the candidate objects within the range $[\alpha, \beta]$, and then computes the aggregate values of these candidate objects with regard to the queried feature x_i. Along with query processing, the server also constructs the verification objects (VOs) for both phases. Once the client receives the query results and VOs, it can authenticate the correctness of the entire query following the verification flow, which is the opposite of the SP's VO construction flow. In what follows, we present the detailed procedure of each phase in this framework, starting with the second phase for clarity reasons.

5.1 Privacy-Preserving Authentication Protocols on Multiset Operations

Before running into the detailed procedure of authenticating aggregate queries, we present five core privacy-preserving authen-

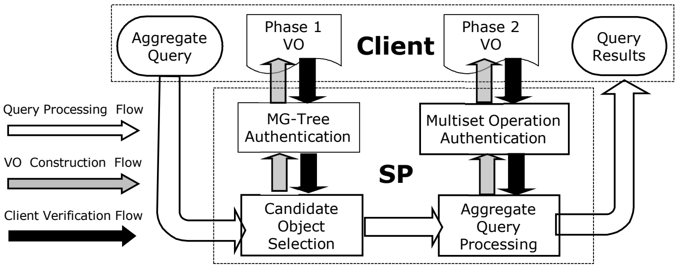


Fig. 2: PA^2 Authentication Framework Overview

tication protocols on multiset operations. The challenge is that the client cannot learn any sensitive feature information when authenticating the multiset operations. Inspired by [27], [30], our key idea is to leverage the randomized bilinear-map accumulator $acc(\cdot)$ (i.e., Eq. (1)) introduced in Section 4 to hash a multiset into a fixed-length value with collision resistance. With the bilinear pairing function $e(\cdot, \cdot)$ (also introduced in Section 4), the client can verify the accumulated value of the output multiset without learning the content of each input multiset. This distinguishes the accumulator function from other cryptographic hash functions such as SHA-1.

In the rest of this section, we introduce the five core privacy-preserving authentication protocols on multiset operations, namely, $subset$, sum, $empty$, $union$, and $times$. For ease of presentation, we mark below any unverified value computed by the SP with *, while all other unmarked values are trusted or already verified by the candidate object selection protocol (to be explained in Section 5.3). We also assume that the seeds $g, g^s, \cdots, g^{s^k}$ are public and that the random values r_{X_i}'s are known to the DO and SP but hidden from the client.

Subset: $sub(X_i, X_j)$. Given two multisets X_i, X_j, it returns the accumulative value $acc(X_j - X_i)$. Note that if $X_i \nsubseteq X_j$, the SP cannot compute a correct value of $acc(X_j - X_i)$. Hence, if $acc(X_j - X_i)$ is verified as correct, the client is assured that $X_i \subseteq X_j$. The detailed protocol is as follows:

i. The SP computes the accumulative value $acc(X_j - X_i)^*$ based on X_i, X_j, the random values r_{X_i}, r_{X_j}, and the public seeds g, g^s, $\cdots$, g^{s^k} (thanks to the nice property of acc as mentioned in Section 4), and sends it to the client together with $acc(X_i)$ and $acc(X_j)$.
ii. The client verifies the correctness of $acc(X_j - X_i)^*$ by checking the following condition, where we exploit the property of bilinear pairing function $e(\cdot, \cdot)$ as mentioned in Section 4.
$$e(acc(X_i), acc(X_j - X_i)^*) \stackrel{?}{=} e(acc(X_j), g).$$

Sum: $sum(\{X_1, \cdots, X_n\})$. Given a set of multisets $\{X_1, \cdots, X_n\}$, it returns the accumulative value $acc(S)$ of the sum set $S = \uplus\{X_i\}$. The detailed protocol is as follows:

i. The SP computes the accumulative values $acc(X_1 \uplus X_2)^*$, $acc(X_1 \uplus X_2 \uplus X_3)^*, \cdots, acc(S)^*$, and sends them to the client together with $acc(X_1), \cdots, acc(X_n)$.
ii. The client verifies the correctness of $acc(S)^*$ by checking the following equations one by one:

$$\begin{cases} e(acc(X_1), acc(X_2)) \stackrel{?}{=} e(acc(X_1 \uplus X_2)^*, g) \\ e(acc(X_1 \uplus X_2)^*, acc(X_3)) \stackrel{?}{=} e(acc(X_1 \uplus X_2 \uplus X_3)^*, g) \\ \vdots \\ e(acc(\uplus_{i=1}^{n-1} X_i)^*, acc(X_n)) \stackrel{?}{=} e(acc(S)^*, g) \end{cases}$$

Empty: $empty(\{X_1, \cdots, X_n\})$. Given a set of multisets $\{X_1, \cdots, X_n\}$, it verifies whether $\cap\{X_i\} = \emptyset$. According to the extended Euclidean algorithm, if $\cap\{X_i\} = \emptyset$, there exist n polynomials Q_i such that $\sum_{i=1}^{n} Q_i \cdot P(X_i) = 1$. The detailed protocol is as follows:

i. The SP computes n values $F_i^* = g^{\frac{Q_i}{r_{X_i}}}$, and sends $F_1^*, \cdots,$ F_n^*, $acc(X_1), \cdots, acc(X_n)$ to the client.
ii. The client verifies the result is empty if the following equation holds:
$$\prod_{i=1}^{n} e(acc(X_i), F_i^*) \stackrel{?}{=} e(g, g).$$

Union: $union(\{X_1, \cdots, X_n\})$. Given a set of multisets $\{X_1, \cdots, X_n\}$, it returns the accumulative value $acc(U)$ of the union set $U = \cup_{i=1}^{n} X_i$. This operation is more complicated than the *sum* operation. Denote $\widehat{X}_i$ as the set version of a multiset X_i. The client needs to verify two conditions:

(1) Deflation checking: $\widehat{X}_1 \subseteq U \wedge \widehat{X}_2 \subseteq U \wedge \cdots \widehat{X}_n \subseteq U$. This is to prevent the SP from deliberately missing any object;
(2) Inflation checking:$(U - \widehat{X}_1) \cap (U - \widehat{X}_2) \cap \cdots (U - \widehat{X}_n) = \emptyset$. This is to prevent the SP from deliberately adding any non-result object.

We can authenticate them based on the above sub and $empty$ protocols as follows:

i. Use protocols $sub(\widehat{X}_1, U)$, $sub(\widehat{X}_2, U)$, $\cdots$, $sub(\widehat{X}_n, U)$ to verify the first condition. This not only authenticates $\widehat{X}_i \subseteq U$, but also provides the client the correct $acc(U - \widehat{X}_i)$.
ii. The SP computes a special hash value $h_e(U)^*$ of the union set U, and by verifying this value the client is assured that the SP knows the pre-image U besides $acc(U)^*$.
iii. Use protocol $empty(\{(U - \widehat{X}_1), (U - \widehat{X}_2), \cdots, (U - \widehat{X}_n)\})$ to verify the second condition.

In step (ii), $h_e(\cdot)$ is an *extractable collision resistant hash* (ECRH) function that has one unique property — extractability [36]. By presenting a correct $h_e(U)^*$ value, the SP can prove that it knows U. Inspired by [36], we design $h_e(\cdot)$ for a multiset $X = \{x_1, x_2, \cdots, x_t\}$ as follows:
$$h_e(X) = (h_1, h_2) = (\prod_{i \in [t]} g^{s^i a_i}, \prod_{i \in [t]} g^{\alpha s^i a_i}),$$
where $s, \alpha \in \mathbb{Z}_p^*$ are secret values only known to the DO of X, and a_i is the i^{th} coefficient for polynomial $P(X) \cdot r_X$. Before any other party can compute or verify $h_e(X)$, the DO releases $g, g^s, \cdots, g^{s^k}, g^{\alpha}, g^{\alpha s}, \cdots, g^{\alpha s^k} \in \mathbb{G}$ to the public, where $k \geq |X|$, the cardinality of X. Then any party, including the client, can verify whether a hash value $h_e(X)$ is correct by checking the following equations:
$$e(h_e(X).h_1, g^{\alpha}) \stackrel{?}{=} e(h_e(X).h_2, g),$$
$$h_e(X).h_1 \stackrel{?}{=} acc(X).$$

Times: $times(X, t)$. Given a multiset X and a coefficient t, it returns the accumulative value $acc(t \cdot X)$ of $t \cdot X$, which raises the multiplicity of each element in X by t. For example, if $X = \{(a, 2), (b, 3)\}$, then $3 \cdot X = \{(a, 6), (b, 9)\}$. A straightforward solution is to use the sum protocol directly as $sum(\{\overbrace{X, X, \cdots, X}^{t}\})$. Here we give a more efficient solution using sum on the binary components. Let $t = (b_0 b_1 \cdots b_d)_2$, the binary form of t. The client only needs to execute $sum(\{b_0 \cdot X, \cdots, b_i \cdot 2^i \cdot X, \cdots, b_d \cdot 2^d \cdot X\})$, where $b_i = 1$. For instance, if

$t = 5 = (101)_2$, then the client only executes $sum(\{X, 4 \cdot X\})$. The detailed protocol is as follows:

i. Let $d = \lfloor log_2(t) \rfloor$, the SP computes $acc(2 \cdot X)^*$, $\cdots$, $acc(2^d \cdot X)^*, acc(t \cdot X)^*$, and sends them to the client together with $acc(X)$.
ii. The client first verifies the values $acc(2 \cdot X)^*, \cdots, acc(2^d \cdot X)^*$ by checking the following equations:

$$\begin{cases} e(acc(X), acc(X)) \overset{?}{=} e(acc(2 \cdot X)^*, g) \\ e(acc(2 \cdot X)^*, acc(2 \cdot X)^*) \overset{?}{=} e(acc(4 \cdot X)^*, g) \\ \vdots \\ e(acc(2^{d-1} \cdot X)^*, acc(2^{d-1} \cdot X)^*) \overset{?}{=} e(acc(2^d \cdot X)^*, g) \end{cases}$$

iii. The client then verifies $acc(t \cdot X)^*$ according to the binary form of $t(b_0 b_1 \cdots b_d)$.

We will show in the cost analysis that this protocol significantly improves the performance over the straightforward sum solution.

5.2 Privacy-Preserving Authentication Algorithms on Aggregate Queries

Next, we present the detailed procedures of the second phase, i.e., authenticating aggregate queries while preserving data confidentiality. Here, we assume that the output from the first phase of candidate object selection has been verified (to be explained in Section 5.3). In what follows, $\{X_1, \cdots, X_m\}$ are the feature sets of m candidate objects, and $S = \uplus\{X_i\}$ is the sum set. We first study the *sum/count* and *max/min/FFQ/top-k* aggregate queries and then extend them to other more advanced aggregate queries.

5.2.1 Sum/Count Query

The output of a sum query $sum(x_q)$ is η_q, the sum of multiplicities of feature x_q in all candidate objects. To align with other multiset queries, we define its result as $R = \{(x_q, \eta_q)\}$. For example, in Figure 1, after selecting the candidate objects $\{o_5, o_6, o_9, \cdots, o_{12}\}$, the query $Q = (sum, \{a\}, [\alpha, \beta])$ returns the sum of multiplicities of feature a, i.e., $R = \{(a, 6)\}$.

To verify this result R, we design Algorithm 1 for the client to check the following two conditions:

- Inflation checking (Lines 1–3): $R \subseteq S$. This is to prevent the SP from deliberately increasing the multiplicity of the result;
- Deflation checking (Line 4): $(S - R) \cap R = \emptyset$. This is to prevent the SP from deliberately decreasing the multiplicity of the result.

Algorithm 1 PA^2 Sum ($\{R, X_1, \cdots, X_m\}$)

```
1: Obtain $R, acc(X_1), \cdots, acc(X_m)$ from the SP (these $acc$
   values are authenticated by the MG-tree, to be detailed in
   Section 5.3) and compute $acc(R)$ locally;
2: Execute $sum(\{X_1, \cdots, X_m\})$ to get verified $acc(S)$;
3: Execute $sub(R, S)$ to get verified $acc(S - R)$; // implies
   $R \subseteq S$
4: Execute $empty(S - R, R)$ to verify $(S - R) \cap R = \emptyset$.
```

In the running example, the sum set of the candidate objects' features is $S = \{(a, 6), (b, 1), (c, 4), (d, 3), (e, 2)\}$. So the client needs to perform:

- Inflation checking: $\{(a, 6)\} \subseteq \{(a, 6), (b, 1), (c, 4), (d, 3), (e, 2)\}$;
- Deflation checking: $\{(b, 1), (c, 4), (d, 3), (e, 2)\} \cap \{(a, 6)\} = \emptyset$.

Similarly, the output of a $count$ query $count(x_q)$ is the number of the candidate objects that have the query feature x_q. It can be processed similarly to the sum query, except that the multiplicity of each feature is enforced as 1.

5.2.2 Max/Min/FFQ/Top-k Query

The output of a max query is the feature with the highest (i.e., top-1) multiplicity. Let τ denote this multiplicity, and thus the query is equivalent to searching for any feature whose multiplicity is no less than τ. Formally, the query result $R = \pi(\{(x_i, \eta_i) | x_i \in S \wedge \eta_i \geq \tau\})$, where $\pi(\cdot)$ randomly selects one feature when there are ties. For example, in Figure 1, the query $Q = (max, \text{-}, [\alpha, \beta])$ returns the result $R = \{(a, 6)\}$. To verify this result, we design Algorithm 2 for the client to check the following three conditions:

- Inflation checking (Lines 1–3): $R \subseteq S$.
- Deflation checking (Line 4): $(S - R) \cap R = \emptyset$. [3]
- Completeness checking (Lines 5–8): $(S - R) \subseteq \tau \cdot (U - \widehat{R})$, where $\widehat{R}$ is the set version of multiset R, e.g., $R = \{(a, 6)\}$ and $\widehat{R} = \{(a, 1)\}$. This is to prevent the SP from deliberately missing any feature whose multiplicity is larger than τ.

Algorithm 2 PA^2 Max ($\{R, X_1, \cdots, X_m\}$)

```
1: Obtain $R, acc(X_1), \cdots, acc(X_m)$ from the SP (these $acc$
   values are authenticated by the MG-tree, to be detailed in
   Section 5.3) and compute $acc(R)$ and $acc(\widehat{R})$ locally;
2: Execute $sum(\{X_1, \cdots, X_m\})$ to get verified $acc(S)$;
3: Execute $sub(R, S)$ to get verified $acc(S - R)$; // implies
   $R \subseteq S$;
4: Execute $empty(S - R, R)$ to verify $(S - R) \cap R = \emptyset$ [3];
5: Execute $union(\{X_1, \cdots, X_m\})$ to get verified $acc(U)$;
6: Execute $sub(\widehat{R}, U)$ to get verified $acc(U - \widehat{R})$;
7: Execute $times(U - \widehat{R}, \tau)$ to get verified $acc(\tau \cdot (U - \widehat{R}))$;
8: Execute $sub(S - R, \tau \cdot (U - \widehat{R}))$ to verify $(S - R) \subseteq$
   $\tau \cdot (U - \widehat{R})$.
```

In the running example, the sum set is $S = \{(a, 6), (b, 1), (c, 4), (d, 3), (e, 2)\}$ and the union set is $U = \{(a, 1), (b, 1), (c, 1), (d, 1), (e, 1)\}$. So the client needs to perform:

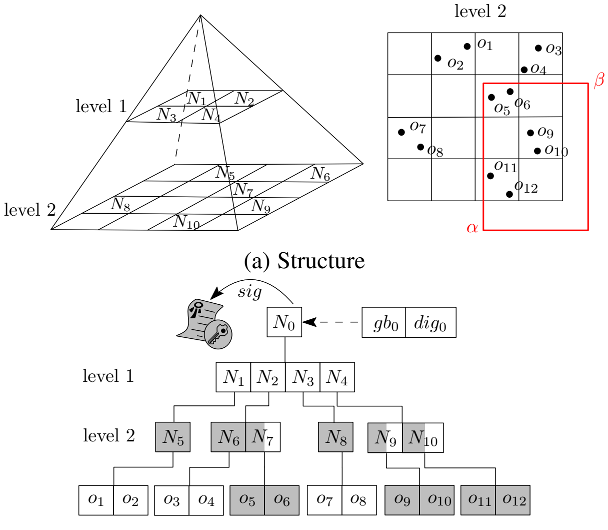
- Inflation checking: $\{(a, 6)\} \subseteq \{(a, 6), (b, 1), (c, 4), (d, 3), (e, 2)\}$;
- Deflation checking: $\{(b, 1), (c, 4), (d, 3), (e, 2)\} \cap \{(a, 6)\} = \emptyset$; [3]
- Completeness checking: $\{(b, 1), (c, 4), (d, 3), (e, 2)\} \subseteq \{(b, 6), (c, 6), (d, 6), (e, 6)\}$.

The min query is similar except that we verify $(S - R) \supseteq \tau \cdot (U - \widehat{R})$ in the completeness checking.

The *FFQ* query returns the features whose multiplicities are no lower than a threshold δ. This query is naturally supported as it is a subroutine of the max query (by replacing the threshold τ with δ in the completeness checking) .

Similarly, the top-k query can be verified by exploiting the routine of max query and replacing the top-1 multiplicity τ with the top-k^{th} multiplicity.

[3] In the actual implementation, the deflation check can be omitted for max, FFQ and top-k queries. This is because it is implied by the following completeness check.



(a) Structure

(b) Index

Fig. 3: Merkle Grid Tree (MG-tree)

5.2.3 Extension to Advanced Aggregate Queries

By fusing multiple primitive aggregate queries, we can further extend our algorithms to support more advanced aggregate queries.

Average Query. The *avg* query returns the average multiplicity of a queried feature. Its result can be authenticated with a sum query and a $count$ query.

Confidence Query. The *confidence* query takes two features as input and returns the conditional probability of witnessing the second feature given the first feature. To revisit Example 1, the confidence that gene 'R-G1886S' co-exists with gene 'C-R102G' is $\frac{2}{4} = 50\%$. This query can be decomposed into two *count* queries for authentication, i.e., $(count,\ \{R\text{-}G1886S\}, [00000,99999])$ and $(count, \{R\text{-}G1886S,\ C\text{-}R102G\}, [00000,99999])$.

Lift Query. The *lift* query is often used to observe the independence between two features. Given two features x_1 and x_2, it is defined as the ratio of $count(\{x_1, x_2\})$ to $count(x_1) \cdot count(x_2)$. In the above example, the lift of gene 'R-G1886S' and gene 'C-R102G' is $\frac{2}{4\times 3} = \frac{1}{6}$. Similar to the *confidence* query, its result can be authenticated with three *count* queries, i.e., $(count,\ \{R\text{-}G1886S\}, [00000,99999])$, $(count,\ \{C\text{-}R102G\}, [00000,99999])$, and $(count, \{R\text{-}G1886S,\ C\text{-}R102G\}, [00000,99999])$.

It is noteworthy that as authenticating the above advanced aggregate queries is always reduced to authenticating a set of primitive aggregate queries, the results of those primitive queries are needed by the client to complete the verification chain and are thus not kept confidential. Nevertheless, the confidentiality of source data is still preserved by the privacy-preserving authentication algorithms. Finally, we remark that a multi-feature aggregate query can be supported by treating a multi-feature set as a unique (virtual) feature.

5.3 Privacy-Preserving Authentication on Candidate Object Selection

Now we turn back to the first phase, i.e., processing and authenticating the candidate object selection. Without loss of generality, we assume that the objects are indexed by a grid structure on their non-sensitive attributes. Thus, we propose *Merkle Grid tree* (MG-tree), an authenticated data structure (ADS) for the DO to construct and sign. Note that our technique can be easily adapted to other multi-dimensional index structures such as R-tree and kd-tree.

Figure 3a shows a multi-layer grid system, which partitions the space of non-sensitive attributes recursively into multiple levels of grid cells until each cell contains no more than two objects. The bounding box of each cell is called a grid box, and is denoted by gb. Figure 3b shows the corresponding MG-tree for the objects in Figure 3a. Every node N_i corresponds to a non-empty grid cell. Its grid box is denoted by gb_i, and has a digest (denoted by dig_i) that is computed from its C child entries, c_1, ..., c_C, as follows:

Definition 1 (Digest for Non-Leaf Node). Let $h(\cdot)$ be a cryptographic hash function and '|' denote string concatenation. The digest of a non-leaf node is defined as:

$$dig_i = h(gb_{c_1}|dig_{c_1}|\cdots|gb_{c_C}|dig_{c_C}).$$

For example, in Figure 3b, the digest of node N_2, $dig_2 = h(gb_6|dig_6\ |gb_7|dig_7)$.

Now we define the digest of a leaf node based on the acc values of leaf entries.

Definition 2 (Digest for Leaf Node). The digest of a leaf node is defined as:[4]

$$dig_i = h(h(acc(X_{c_1}))|\cdots|h(acc(X_{c_C}))),$$

where $acc(X_i)$ is the accumulative value of the feature set of object X_i (defined in Eq. (1)).

For example, for leaf node N_5, its digest $dig_5 = h(h(acc(X_1))\ |h(acc(X_2)))$. Note that this digest does not need any information about objects' non-sensitive attributes A_i, because the digest of its parent node already proves that this object is in its grid box.[5] With these definitions, the DO can compute the acc values of all feature sets and the digests of all nodes in the MG-tree recursively in a bottom-up fashion. The digest of the root entry dig_0 is signed by the DO as $sig(dig_0)$.

On the SP side, the processing of a range query $[\alpha, \beta]$ starts from the root node. If a non-leaf node intersects the query range $[\alpha, \beta]$, it will be branched, i.e., its subtree is further explored. On the other hand, if a leaf node intersects the query range $[\alpha, \beta]$, all its entries will be returned as query results. Figure 3a illustrates an example where $N_0, N_1, \cdots, N_{10}$ are nodes and $o_1, o_2, \cdots, o_{12}$ are objects, i.e., leaf entries. Since the root node N_0 intersects with the range $[\alpha, \beta]$, it will be branched and then non-leaf nodes N_2, N_4 are also branched. After that, leaf nodes N_7, N_9, N_{10} intersect the query range, and all the leaf entries $\{o_5, o_6, o_9, o_{10}, o_{11}, o_{12}\}$ are returned as results, i.e., the selected candidate objects for the next phase of aggregate query processing.

To guarantee the correctness of the selected candidate objects, the query authentication protocol on the MG-tree is as follows:

- The SP prepares a VO and sends it to the client. The VO includes: (i) the grid boxes of all intersected leaf nodes and the accumulative acc values of their leaf entries; (ii) the grid boxes and the digests of all other visited (but not intersected by the query) nodes; (iii) the signature of the root's digest. For example, in Figure 3b, the VO (marked as grey) includes $\{gb_5, dig_5, gb_6, dig_6, gb_7, acc(X_5), acc(X_6), gb_8,$

[4] In fact, the digest also includes $h(acc(\widehat{X}_{c_1}))|\cdots|h(acc(\widehat{X}_{c_C}))$, where $\widehat{X}_{c_i}$ is the set version of a multiset X_{c_i}. These acc values are needed for the $union(\cdot)$ protocol in Section 5.1 but omitted for clarify of presentation.

[5] For simplicity, in this paper we assume that the query range aligns with the grid boxes. Otherwise, the definition can be easily extended to include A_i in the digests of leaf nodes.

dig_8, gb_9, $acc(X_9)$, $acc(X_{10})$, gb_{10}, $acc(X_{11})$, $acc(X_{12})$, $sig(dig_0)\}$.

- The client first verifies the correctness of the returned results, that is, the grid boxes of all the result nodes are intersected with the query range while all the others are not. Then, the client verifies whether all these information is genuine by restoring the root digest. Specifically, the client recursively rebuilds the digests of leaf nodes and non-leaf nodes up to the root level. Finally, the client checks whether the restored digest matches the signature from the DO. If so, the client verifies that all the returned information is genuine. For example, in Figure 3b, the client first verifies gb_7, gb_9, gb_{10} are intersected with the query range $[\alpha, \beta]$, while gb_5, gb_6, gb_8 are not. Next, the client rebuilds $dig_7 = h(h(acc(X_5))| h(acc(X_6)))$, $dig_9 = h(h(acc(X_9))|h(acc(X_{10})))$, $dig_{10} = h(h(acc(X_{11}))|h(acc(X_{12})))$, and then $dig_1 = h(gb_5 |dig_5), dig_2 = h(gb_6|dig_6|gb_7|dig_7), dig_3 = h(gb_8|dig_8)$, $dig_4 = h(gb_9|dig_9|gb_{10}|dig_{10})$. And finally, the client restores $dig_0 = h(gb_1|dig_1| \cdots |gb_4|dig_4)$ and checks whether it matches $sig^{-1}(dig_0)$.

5.4 Cost Analysis

In this section, we analyze the performance of PA^2 framework. Specifically, we derive cost models for the privacy-preserving authentication protocols on multiset operations, the aggregate query authentication algorithms, and the MG-tree, respectively.

5.4.1 Cost Model for Authentication Protocols on Multiset Operations

Table 3 summarizes the time costs for the SP and client, as well as the VO size for the five core privacy-preserving authentication protocols on multiset operations (Section 5.1). In this table, n is the total number of objects, t is the second operand for $times(\cdot, \cdot)$, C_e is the time cost of a bilinear pairing operation, C_{gp} is the time cost of a multiplication operation in cyclic multiplicative group $\mathbb{G}$ with order p, $|U|$ is the size of union set, and M_{acc} is the size of an accumulative value.

5.4.2 Cost Model for Aggregate Query Authentication Algorithms

Based on the above analysis, we now derive the cost models for various aggregate queries. Let m be the number of candidate objects, $|S|$ be the size of the sum set, $|\widehat{R}|$ be the size of set $\widehat{R}$, $|X_i|$ be the size of the feature set for the i^{th} candidate object, and τ be the top-k^{th} (top-1 for a *max* query) multiplicity.

Count/Sum Aggregate Query. The time cost for the SP is:

$$\begin{aligned} C^S_{count} &= C^S_{sum} + C^S_{sub} + C^S_{empty} \\ &= (\sum_{i=1}^{m}(i \cdot |X_i|) + |S| - |R| \\ &\quad + \max(|S - R|, |R|)) \cdot log_2(p) \cdot C_{gp}. \end{aligned}$$

The time cost for the client is:

$$\begin{aligned} C^C_{count} &= C^C_{sum} + C^C_{sub} + C^C_{empty} + |R| \cdot log_2(p) \cdot C_{gp} \\ &= (2m + 1) \cdot C_e + |R| \cdot log_2(p) \cdot C_{gp}. \end{aligned}$$

The VO size, i.e., the communication cost, is:

$$\begin{aligned} M_{count} &= M_{sum} + M_{sub} + M_{empty} \\ &= (4m + 3) \cdot M_{acc}. \end{aligned}$$

Max/Min/FFQ/Top-k Aggregate Query. The time cost for the SP is:

$$\begin{aligned} C^S_{max} &= C^S_{sum} + 3C^S_{sub} + C^S_{empty} + C^S_{union} + C^S_{times} \\ &= (\sum_{i=1}^{m}(i \cdot |X_i|) + (m + 2 + 2\tau)|U| + max(|S - R|, |R|) \\ &\quad + max_{i=1}^{n}|\widehat{X}_i| - (1 + \tau)|\widehat{R}| - \sum_{i=1}^{n}|\widehat{X}_i|) \cdot log_2(p) \cdot C_{gp}. \end{aligned}$$

The time cost for the client is:

$$\begin{aligned} C^C_{max} &= C^C_{sum} + 3C^C_{sub} + C^C_{empty} + C^C_{union} + C^C_{times} \\ &\quad + (|R| + |\widehat{R}|) \cdot log_2(p) \cdot C_{gp} \\ &= (3m + 6 + 2log_2(\tau)) \cdot C_e + (|R| + |\widehat{R}|) \cdot log_2(p) \cdot C_{gp}. \end{aligned}$$

The VO size, i.e., the communication cost, is

$$\begin{aligned} M_{max} &= M_{sum} + 3M_{sub} + M_{empty} + M_{union} + M_{times} \\ &= (7m + 14 + 2log_2(\tau)) \cdot M_{acc}. \end{aligned}$$

A key observation from the above analysis is that both the client time cost and the VO size are determined only by the result set size and are independent of the number of sensitive features.

5.4.3 Cost Model for Candidate Object Selection

We next analyze the cost of the candidate object selection. Our analysis starts with the MG-tree size, based on which we derive the VO construction and verification costs for a range selection. Without loss of generality, the data space is a d-dimensional unit space $[0, 1]^d$, and we assume each non-sensitive attribute independently follows a uniform distribution. As such, the MG-tree partitions each dimension equally and the height of the tree can be approximated as $w = \lceil log_{2^d}(\frac{n}{f})\rceil$, where n is the total number of objects and f is the minimum number of objects in a leaf node.

MG-tree size. Each tree node stores a grid box (represented by two d-dimensional corner points) and a digest. As such, the size of a node, in terms of bytes, is:

$$M_N = 2 \cdot d \cdot 4 + M_h,$$

where M_h is the size of a hash value.

Besides the tree nodes, the server stores all leaf entries (i.e., objects). Let M_{acc} denote the size of an accumulative value; then the total size of an MG-tree is:

$$M_{MG} = 2 \cdot n \cdot M_{acc} + \sum\nolimits_{i=0}^{w-1} min(n, 2^{i \cdot d}) \cdot M_N.$$

Construction cost of VO. According to [37], the probability that two random rectangles R_1, R_2 overlap is:

$$Pr_{overlap}(R_1, R_2) = \prod\nolimits_{j=1}^{d}(R_1.L_j + R_2.L_j), \qquad (2)$$

where $R_i.L_j$ is the length of R_i in dimension j. Since both the MG-tree partition and object distribution are uniform, the lengths of an intermediate entry at level i and a leaf entry are 2^{-i} and $\sqrt[d]{k/n}$, respectively. Putting them into Eq. (2), the numbers of visited tree nodes N_n and visited leaf entries N_e are:

$$N_n = \sum_{i=0}^{w-1} 2^{i \cdot d} \prod_{j=1}^{d}(2^{-i} + Q.L_j),$$

$$N_e = n \cdot \prod_{j=1}^{d}(\sqrt[d]{\frac{k}{n}} + Q.L_j),$$

respectively. Let $C_{\otimes}$ denote the time cost of accessing a single entry or a single node. The cost of constructing the VO is modeled as:

$$C_{pre} = (N_n + N_e) \cdot C_{\otimes}.$$

Operation	SP Time	Client Time	VO Size
$sub(X_1, X_2)$	$C^S_{sub} = (\|X_2\| - \|X_1\|) \cdot log_2(p) \cdot C_{gp}$	$C^C_{sub} = C_e$	$M_{sub} = 3M_{acc}$
$sum(\{X_1, \cdots, X_n\})$	$C^S_{sum} = \sum_{i=1}^n (i \cdot \|X_i\|) \cdot log_2(p) \cdot C_{gp}$	$C^C_{sum} = n \cdot C_e$	$M_{sum} = 2n \cdot M_{acc}$
$empty(\{X_1, \cdots, X_n\})$	$C^S_{empty} = \max_{i=1}^n (\|X_i\|) \cdot log_2(p) \cdot C_{gp}$	$C^C_{empty} = n \cdot C_e$	$M_{empty} = 2n \cdot M_{acc}$
$union(\{X_1, \cdots, X_n\})$	$C^S_{union} = ((n+1) \cdot \|U\| - \sum_{i=1}^n \|\widehat{X}_i\|$ $+ \max_{i=1}^n \|\widehat{X}_i\|) \cdot log_2(p) \cdot C_{gp}$	$C^C_{union} = (2n+1) \cdot C_e$	$M_{union} = (5n+1) \cdot M_{acc}$
$times(X, t)$	$C^S_{times} = 2t \cdot \|X\| \cdot log_2(p) \cdot C_{gp}$	$C^C_{times} = 2log_2(t) \cdot C_e$	$M_{times} = 2log_2(t) \cdot M_{acc}$

TABLE 3: Cost Models for Multiset Operations

VO size (communication cost). A VO includes three parts: (i) the grid boxes of all intersected leaf nodes and accumulative values of leaf entries, i.e., $\frac{N_e}{k} \cdot 8 \cdot d + N_e \cdot M_{acc}$; (ii) the grid boxes and digests of all boundary nodes, i.e., $N_e \cdot w \cdot (2^d - 1)$; (iii) the signature of tree root's digest, i.e., M_{sig}. As such, the communication cost of the VO is:

$$M_{VO} = 8d\frac{N_e}{k} + N_e \cdot M_{acc} + N_e \cdot w(2^d - 1)(8d + M_h) + M_{sig}.$$

Client verification time cost. The client verifies the VO by first reconstructing the MG-tree, and then checks the signature:

$$C_{ver} = (N_n + N_e) \cdot C_h + C_{sig},$$

where C_h, C_{sig} are the time costs of calculating a hash value and verifying a signature, respectively.

6 Security Analysis

In this section, we perform a security analysis on our PA^2 algorithms for aggregate queries.

6.1 Security of PA^2 Algorithms

The correctness of our PA^2 algorithms is defined in the natural way and is omitted. For soundness, it can be formally defined as follows:

Definition 3 (Soundness). The PA^2 authentication algorithms for aggregate queries are sound if for all PPT adversaries, the success probability is negligible in the following experiment:

- The adversary picks a dataset $\mathbb{D}$.
- Run the ADS generation on $\mathbb{D}$ and forward to the adversary.
- The adversary outputs a query Q, a result R, and a VO.

We say the adversary succeeds if the VO passes the result verification and $R \neq Q(\mathbb{D})$.

We now show that the PA^2 authentication algorithms indeed satisfy the desired security requirements.

Theorem 1. The PA^2 authentication algorithms for aggregate queries guarantee the correctness and soundness of the query results under the bilinear q-strong Diffie-Hellman assumption.

Proof. The theorem is proved by proving the security of each multiset operation: $sub(\cdot, \cdot)$, $empty(\cdot)$, $sum(\cdot)$, $union(\cdot)$, and $times(\cdot, \cdot)$. See Appendix A in the Supplemental Material for more detailed proofs. □

6.2 Privacy Guarantee on Sensitive Features

Next, we analyze the privacy guarantee on the sensitive features in our PA^2 framework. Specifically, we show that the accumulative value of a feature set does not disclose any feature information to the client. Formally, it is stated as follows:

Definition 4 (IND-CPA). The augmented accumulative value $acc(X)$ for a multiset X is indistinguishable if for all PPT adversaries, there exists a negligible probability $neg(q)$, s.t.

$$Pr[\mathsf{Adv} \to \{X_1, X_2, st\}; b \to \{0, 1\};$$
$$acc(X_b) \to c; \mathsf{Adv}(st, c) \to b'; b = b'] = \frac{1}{2} + neg(q).$$

Before showing the indistinguishability of accumulative values, we start with the following lemma on the group $\mathbb{G}$.

Lemma 1. For any random value $r \leftarrow \mathbb{Z}_p$, g^r has an equal probability of being any element in $\mathbb{G}$ (with an order of p). Formally, for any $\hat{g} \in \mathbb{G}$,

$$Pr[g^r = \hat{g}] = 1/p.$$

Proof. Let $log_g()$ denote the discrete logarithm of base g in group $\mathbb{G}$. We have

$$Pr[g^r = \hat{g}] = Pr[r = log_g(\hat{g})].$$

Since r is random, the probability of r being a fixed element $log_g(\hat{g})$ equals $1/p$. □

Now we show a theorem on the security of accumulative values.

Theorem 2. The augmented accumulative value $acc(X)$ for a multiset X is indistinguishable under chosen plaintext attack (CPA).

Proof. Since the augmented accumulative value $acc(X) = g^{P(X) \cdot r_X} = (g^{P(X)})^{r_X}$, according to the above lemma, $acc(X)$ has an equal probability of being any element in $\mathbb{G}$. As such, the verifier learns nothing about $P(X)$ and any element $x_i \in X$. Further, since r_X is random for each X, $acc(X)$ is indistinguishable under CPA. □

It is worth noting that by achieving indistinguishability on a single accumulative value, multiple accumulative values are automatically guaranteed to be indistinguishable. Therefore, the above theorem effectively proves the security of the authentication algorithms developed for various primitive aggregate queries, as only (indistinguishable) accumulative values are disclosed for the sensitive features throughout the authentication process, which protects the confidentiality of source data.

6.3 Discussion on Privacy Guarantee

It is noteworthy that during the authentication of aggregate queries, the results of primitive aggregate queries are disclosed to the client, which might be a source of privacy leak. In an extreme case, if there is only one candidate object in the selection phase, the query result could directly reveal the features of that object. Further, the difference of two query results could reveal the features of the difference of the two candidate object sets. As more aggregate queries are issued and each may consist of multiple primitive aggregate queries, it becomes easier to find smaller difference of candidate object sets. To mitigate such privacy leak, inspired by the bucketization approach [38], we impose a minimum granularity on the MG-tree leaf nodes for the non-sensitive attributes. Specifically, when building the MG-tree, the DO enforces each leaf node (i.e., a bucket) to contain a minimum number of objects that can satisfy a given privacy metric (e.g., k-anonymity or t-closeness [39], [40]). As such, the data space

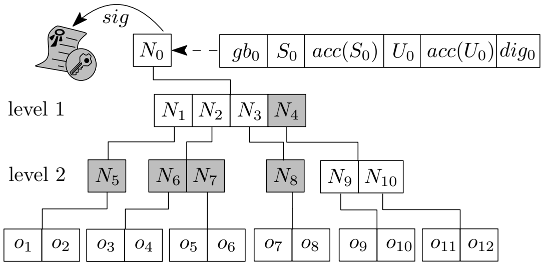


Fig. 4: Optimized MG-tree

is partitioned into disjoint buckets, and upon query processing, a bucket is the smallest granule for range selection. With this enforcement, no query result can reveal whether it is contributed by a specific object in a bucket, and the latter becomes the smallest granule of result privacy leak, no matter how many aggregate queries are issued. Formally, the results of aggregate queries and the associated authentication process achieve the following *bucket-wise indistinguishability* privacy:

Definition 5 (Bucket-wise Indistinguishability). Given a bucket $\mathcal{B}$ and an object $o \in \mathcal{B}$, if an aggregate query result R is from a candidate object set $\mathcal{O} \supseteq \mathcal{B}$, then any adversary can determine whether o contributes to R with a success rate no higher than that of a random guess.

7 Optimizations

This section introduces three optimization techniques for our PA^2 authentication framework. They are orthogonal to each other, and therefore can be combined to further boost the performance.

7.1 Optimized MG-Tree

One performance bottleneck is that the SP has to use the costly exponentiation operation rather than modular exponentiation to compute the accumulative values because it does not know the secret value of s. To alleviate this, we let the DO pre-compute and store some intermediate accumulative values in the MG-tree for the SP. For example, in Figure 3, to authenticate the query $Q = (max, \text{-}, [\alpha, \beta])$, the SP needs to compute the $acc(S)$ value of $S = X_5 \uplus X_6 \uplus X_9 \uplus \cdots \uplus X_{12}$. To do so, the SP has to compute five intermediate accumulative values in turn: $acc(X_5 \uplus X_6), acc(X_5 \uplus X_6 \uplus X_9), \cdots, acc(X_5 \uplus X_6 \uplus X_9 \uplus \cdots \uplus X_{12})$. However, if the DO has already computed $acc(S_7)$ and $acc(S_4)$ for the two intermediate nodes N_7 and N_4, the SP can simply compute $acc(S) = sum(\{S_7, S_4\})$.

As the DO knows the value of s, computing such intermediate accumulative values is efficient. Nonetheless, to alleviate the SP's storage cost, we only compute and store the accumulative values of sum and $union$ operations as they are the most time-consuming operations. To accommodate these intermediate values in the MG-tree, we redesign the digest for non-leaf nodes as follows:

Definition 6 (Optimized Digest for Non-Leaf Node). Let C be the number of child entries of node N_i, S_i be the sum set of child entries' sum sets, and U_i be the union set of child entries' union sets; the digest of node N_i is defined as:

$$dig_i = h(gb_{c_1}|h(acc(S_{c_1})|acc(U_{c_1}))|dig_{c_1}| \cdots \\ |gb_{c_C}|h(acc(S_{c_C})|acc(U_{c_C}))|dig_{c_C}).$$

Based on this optimized MG-tree, the query $Q = (max, \text{-}, [\alpha, \beta])$ no longer needs to traverse down to the child nodes if the current tree node is fully contained in the query range $[\alpha, \beta]$. As such, the candidate object selection and PA^2 algorithms on the optimized MG-tree are revised as follows:

- The SP prepares the VO, which includes (i) the grid boxes, the accumulative values of sum and union sets, and the digests of those fully contained tree nodes; (ii) the grid boxes, the hash values for the accumulative values of sum and union sets, and the digests of all boundary nodes; (iii) the signature of the tree root's digest. For example, in Figure 4, for the same query $Q = (max, \text{-}, [\alpha, \beta])$, the VO (marked as grey) includes $\{gb_5, h(acc(S_5)|acc(U_5)), dig_5, gb_6, h(acc(S_6)|acc(U_6)), dig_6, gb_7, acc(S_7), acc(U_7), dig_7, gb_8, h(acc(S_8)|acc(U_8)), dig_8, gb_4, acc(S_4), acc(U_4), dig_4, sig(dig_0)\}$.
- The client still verifies the VO by reconstructing the root digest of the MG-tree recursively. First, the client verifies that the grid boxes are fully inside or outside the query range. Then, the client rebuilds the digest for each node. Finally, the client checks whether the restored root digest matches the signed one. For example, the client verifies gb_5, gb_6, gb_8 are fully outside the range $[\alpha, \beta]$ and gb_4, gb_7 are fully inside the range. Next, the client rebuilds $dig_1 = h(gb_5|h(acc(S_5)|\ acc(U_5))|dig_5)$, $dig_2 = h(gb_6| h(acc(S_6)|acc(U_6))|dig_6|gb_7|h(acc(S_7)|acc(U_7))|dig_7)$, $dig_3 = h(gb_8|h(acc(S_8)|acc(U_8))|dig_8)$. And then, the client rebuilds $dig_0 = h(gb_1|h(acc(S_1)|acc(U_1))| dig_1| \cdots |gb_4|h(acc(S_4)|acc(U_4))|dig_4)$ and finally compares it against $sig^{-1}(dig_0)$.
- Next, the client proceeds to the PA^2 algorithms with input $\{R, S_1, U_1, \cdots, S_{m'}, U_{m'}\}$. The difference is that the accumulative values of sum and $union$ sets are computed from the intermediate sum and $union$ sets. That is, for a sum operation, the client computes $acc(S) = sum(\{S_1, \cdots, S_{m'}\})$ instead of $sum(\{X_1, \cdots, X_m\})$. Similarly, for a $union$ operation, the client computes $acc(U) = union(\{U_1, \cdots, U_{m'}\})$ instead of $union(\{X_1, \cdots, X_m\})$.

Regarding the performance, since the number of subsets to compute the acc values from is reduced from m to $log(m)$, the SP's computation, the client's computation, and the communication complexity of computing the sum set are reduced from $O(m^2)$, $O(m)$, $O(m)$ to $O(m)$, $O(log(m))$, $O(log(m))$, respectively. Similarly, the costs of computing the $union$ set are reduced from $O(m \cdot (|S| + |U|)), O(m), O(m)$ to $O(log(m) \cdot (|S|+|U|)), O(log(m)), O(log(m))$, respectively. As computing the sum and $union$ sets dominates the total computation and communication complexity of their respective queries, we expect the overall performance gain is significant.

7.2 Accumulating $sum(\cdot)$ by Linear Ordering

For the $sum(\cdot)$ operation, it has a nice additive property that $sum(\{X\}) + sum(\{Y\}) = sum(\{X\} \uplus \{Y\})$. Therefore, we can ask the DO to pre-compute sum accumulative values in an *accumulative* manner. Let us consider the 1D case for simplicity and sort the objects in ascending order. For each $i \in [1, n]$ (n is the number of objects), the DO pre-computes, sum_{1-i}, the sum accumulative value for the objects from o_1 up to o_i. Then, for any range query $[\alpha, \beta]$ that selects the objects from i to j, the sum value sum_{i-j} can be computed as $sum_{i-j} = sub(sum_{1-(i-1)}, sum_{1-j})$. For example, in

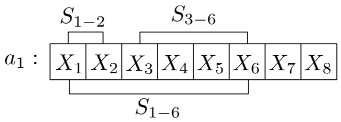


Fig. 5: Sum Optimization

Figure 5, all eight objects are sorted by attribute a_1. The DO pre-computes the sum accumulative values of $sum(\{X_1\})$, $sum(\{X_1, X_2\}), \cdots, sum(\{X_1, \cdots, X_n\})$. As such, the sum accumulative value sum_{3-6} can be computed by $sub(sum_{1-2}$, $sum_{1-6})$.

Regarding the performance, similar to the optimized MG-tree, since the DO knows the secret value of s, the pre-computation is very efficient. As for the query cost, let m denote the number of candidate objects; both the client's computation and communication complexity will be reduced to $O(1)$. While the computation complexity for the SP is still $O(m)$, the constant ratio will be greatly reduced with fewer $sum(\cdot)$ operations. Note that for multidimensional space, we can still enforce linear ordering on the objects and adopt this optimization, by using space embedding techniques such as the Hilbert Curve. Nonetheless, the performance gain is less significant because sum_{i-j} might not be the exact objects selected by a range and some further $subset$ operations would be needed.

7.3 Acceleration by Parallelism

Both the multiset authentication protocols and PA^2 algorithms are highly parallelizable. As such, we can accelerate the authentication performance by embracing parallel computing architectures, such as multithreading, GPU computing, and MapReduce. For example, in the $union(\cdot)$ protocol, we can map all $sub(\cdot, \cdot)$ jobs to the available computing units, whether they are CPU cores or worker nodes in MapReduce. Similarly, in the $empty(\cdot)$ protocol, the server can compute the accumulative values of polynomials Q_i in a parallel manner. Note that, however, in the $sum(\cdot)$ and $times(\cdot, \cdot)$ protocols, the order of polynomials is accumulated and therefore the time cost of each parallel job may vary significantly. As such, instead of splitting all jobs equally at the beginning of the query, we schedule a small set of jobs at a time. With this graceful scheduling technique, we can minimize the waiting time of idle computing units while still reducing the job scheduling overhead.

8 Performance Evaluation

In this section, we evaluate the performance of PA^2 framework for aggregate queries over set-valued data, including the *max*, *top-k*, *sum*, and *FFQ* queries. Three datasets are used in the experiments, namely *Personal Genome Project at Harvard Medical School* (PGP) [2], *Foodmarket from Microsoft* (FoodMarket) [41], and *TPC Benchmark H* (TPC-H) [42]. As introduced in Section 1, the PGP dataset contains personal genome data of 600 participants. The objects in this dataset are in the form of <$latitude$, $longitude$, $mut\text{-}genes$>, where $mut\text{-}genes$ is a feature set of mutation genes of an individual, e.g., {'A-C130R', 'P-I696M'}. There are totally 395 unique mutation genes, and on average each individual has 35 of them. The FoodMarket is a larger dataset that contains $164,558$ shopping transaction records from $8,842$ users and $1,560$ products, in the form of <$birthday$, $membershipAge$, $items$>. Finally, the TPC-H dataset contains $1,020,116$ transaction records from $255,000$ orders and $1,700$ suppliers, in the form of <$order\_priority$, $date$, $suppliers$>. In all these three datasets, the first two attributes are non-sensitive, whereas the last attribute is sensitive. We use the FoodMarket as the default dataset unless otherwise stated. For the experiments requiring higher dimensionality, we also synthesize some attributes into this dataset.

Algorithm	SP CPU Time (s) # Features per Multiset			Client CPU Time (ms)	VO Size (Bytes)
	100	1,000	10,000		
$sub(\cdot, \cdot)$	0.03	0.5	16	2	384
$sum(\cdot)$	0.04	0.4	3.6	5	384
$union(\cdot)$	0.1	1.3	33	20	1,024
$empty(\cdot)$	0.04	0.5	16	5	512
$times(\cdot, 10)$	0.4	4.1	43	20	640

TABLE 4: Performance vs. # Features

Algorithm	SP CPU Time (s) # Multisets			
	5	10	20	50
$sum(\cdot)$	0.1	0.3	1.4	8.3
$union(\cdot)$	0.4	2.6	18	240
$empty(\cdot)$	0.01	0.01	0.01	0.01

TABLE 5: Performance vs. # Multisets

Both the DO and the query client are set up on a commodity laptop computer with Intel Core i5 CPU and 4GB RAM, running on macOS Sierra. The SP is set up on an x64 blade server with dual Intel Xeon 2.2GHz E5-2630 CPU and 256GB RAM, running on CentOS 6. The experiments are written in C++ and use the following libraries: Pairing-based cryptography for bilinear pairing computations, Flint for modular arithmetic operations, Crypto++ for 160-bit SHA-1 hash operations, OpenMP for parallel computation.

To evaluate the performance of the authentication process, we mainly measure two costs: (i) the computational cost in terms of the SP and client CPU time, and (ii) the communication cost in terms of the size of data transmitted (a) between the DO and the SP (i.e., the index size) and (b) between the SP and the client (i.e., the VO size). The results are reported based on an average of 10 randomly generated operations/queries.

8.1 Performance of Privacy-Preserving Multiset Authentication Protocols

We first use two synthetic multisets to evaluate the performance of the five privacy-preserving authentication protocols, i.e., $sub(\cdot, \cdot)$, $sum(\cdot)$, $union(\cdot)$, $empty(\cdot)$, and $times(\cdot, \cdot)$ in terms of (i) the SP CPU time, (ii) the client CPU time, and (iii) the VO size. We fix the coefficient for $times(\cdot, \cdot)$ to 10, and vary the number of features from 100 to $10,000$. The SP CPU time (single-threaded) is reported in Table 4. As coinciding with our cost model analysis, the SP CPU time is almost linear to the number of features. In contrast, the client CPU time and VO size are all constants, irrespective of the number of features.

We further investigate the SP CPU time of $sum(\cdot)$, $union(\cdot)$, $empty(\cdot)$ for more than two input multisets. We fix the number of features in each multiset to 20 and vary the number of multisets. Since the client time and VO size are both linear to the number of input multisets, Table 5 only shows the SP CPU time. For the $sum(\cdot)$ and $union(\cdot)$ operations, the cost is almost quadratic to the number of multisets, which coincides with our analysis in Table 3. For $union(\cdot)$, we also investigate whether the cardinality of feature universe affects its performance. Table 6 shows the

Algorithm	SP CPU Time (s) # Unique Features			
	50	100	200	500
$union(\cdot)$	0.14	0.26	0.33	0.39

TABLE 6: Performance of $union(\cdot)$ vs. # Unique Features

Dataset	Dataset Size (MB)	Setup Time (s)	MG-tree Index Size (MB)
PGP	0.08	9.7	0.42
FoodMarket	0.9	136	7.1
TPC-H	13	1,365	116

TABLE 7: DO Setup Overhead

results as the features increase while fixing the number of multisets and the multiset size to 5 and 20, respectively. We observe that the increment is sublinear to the cardinality of feature universe.

8.2 Query Authentication Performance

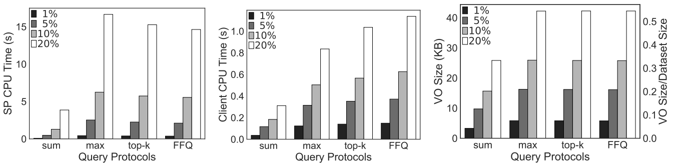
We turn on all the optimizations and evaluate the overall performance of privacy-preserving query authentication on all three datasets. First of all, Table 7 shows the DO setup cost for generating the acc values and constructing the MG-tree. As explained in the cost model analysis, since the DO knows the secret value of s, the setup process is relatively efficient.

To select the candidate objects, we choose the participants' location attributes in PGP, the customers' birthday and membership age in FoodMarket, and the order_priority and date in TPC-H as the input attributes for 2D range queries. The aggregate queries, *max*, *top-k*, *sum*, and *FFQ*, are then performed on the feature sets of the selected candidate objects. For *top-k* queries, we set $k = 5$; and for *FFQ* queries, we set the threshold to be 80% of the highest multiplicity of all features. The experiments are conducted under a multi-threaded setting (40 hyper-threads on 20 CPU cores). The results are shown in Figure 6, Figure 7, and Figure 8 respectively, where the query range varies from 1% to 20% of the data space. It is observed that all the costs are generally sublinear to the query range, thanks to the optimized MG-tree that significantly reduces the multiset operations and accumulative value computations. It is interesting to note that the cost of client CPU time is relatively higher for *FFQ* queries on TPC-H dataset. This is caused by the large multiplicity of the returned multiset result, which incurs more overhead when computing the result acc value on the client. Nevertheless, the consistent results on the three different datasets suggest that our algorithms are robust against various data distributions.

For *FFQ* queries, we also compare our proposed privacy-preserving authentication algorithm with the state-of-the-art non-privacy-preserving counterpart proposed in [28], denoted by *FFQ-NP*. This comparison intends to show the overhead incurred by the confidentiality preservation requirement. We select 1% of the PGP data as input and set the maximum size of frequent feature sets to 3. As shown in Table 8, the overhead is moderate in that our *FFQ* is only tens of times worse than *FFQ-NP* in terms of the DO index size and SP CPU time. The VO size of our *FFQ* is even smaller than that of *FFQ-NP*. Another interesting observation is that the SP CPU time is positively correlated to the threshold δ in our algorithm, but negatively in *FFQ-NP*. The is because the $times(\cdot, \cdot)$ operation involved in *FFQ*, whose input is the infrequent feature sets, becomes more costly with increasing threshold. In comparison, the SP CPU time of *FFQ-NP* is mainly affected by the number of frequent feature sets, which decreases with a larger threshold. As *FFQ* queries are often used in the early stage of the whole data analytics pipeline, their threshold is usually set to a low or medium value, which falls in the comfort zone of our algorithm.

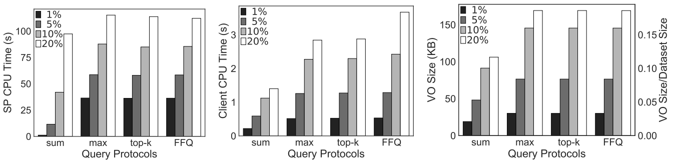
Algorithm	DO Setup Time (s)	DO Index Size (KB)	δ	SP CPU Time (s)	Client CPU Time (s)	VO Size (KB)
FFQ	8.9	307	50%	24	6.1	1.1
			70%	25	1.1	1.1
			90%	45	0.4	1.1
FFQ-NP	0.1	13.7	50%	15.1	22.0	1668
			70%	5.0	7.0	346
			90%	2.5	1.8	88

TABLE 8: Proposed *FFQ* Algorithm vs. *FFQ-NP* Algorithm



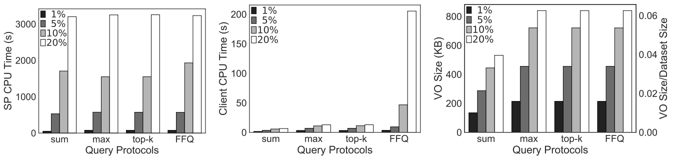
(a) SP CPU Time (b) Client CPU Time (c) VO Size

Fig. 6: Query Performance vs. Selectivity on PGP dataset



(a) SP CPU Time (b) Client CPU Time (c) VO Size

Fig. 7: Query Performance vs. Selectivity on FoodMarket dataset



(a) SP CPU Time (b) Client CPU Time (c) VO Size

Fig. 8: Query Performance vs. Selectivity on TPC-H dataset

8.3 Impact of Optimizations

To evaluate the impact of the proposed optimizations, namely, the optimized MG-tree (OMG) and parallelizing techniques, we use the FoodMarket dataset and set the selectivity of candidate objects to 5% of the whole dataset. Their effects for different aggregate queries are shown in Figure 9. It can be seen that both the OMG-tree and multithreading techniques substantially reduce the SP CPU time by an order of magnitude. In Figure 10, we further vary the number of threads in parallelization. We observe that the SP can be effectively accelerated by up to 30-40 threads, only bounded by the number of logical CPUs of our server. As we show the computation of the SP is highly parallellizable, we believe its performance can be further accelerated by clusters or cloud IaaS widely available in today's data centers.

We also evaluate the performance of the linear ordering technique in Figure 11. We compare it with the baseline $sum(\cdot)$ under one-dimensional data. The results show that it dramatically improves the performance by more than four times. In addition, as the selectivity increases, the performance gain becomes even more eminent.

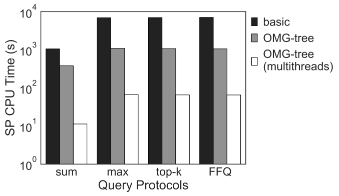


Fig. 9: Performance vs. Optimizations

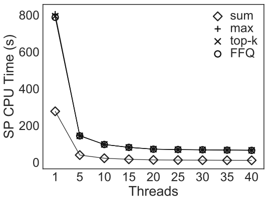


Fig. 10: Multi-Threaded Acceleration

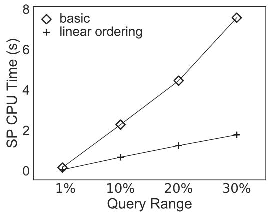
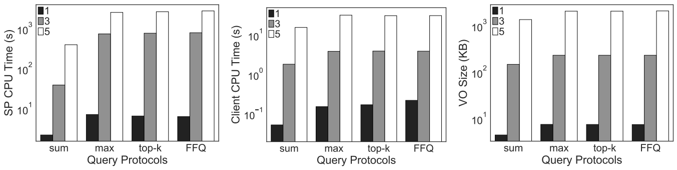
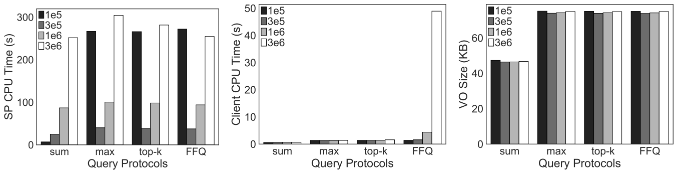


Fig. 11: Basic $sum(\cdot)$ vs. Linear Ordering



(a) SP CPU Time (b) Client CPU Time (c) VO Size

Fig. 12: Query Performance vs. Dimensionality



(a) SP CPU Time (b) Client CPU Time (c) VO Size

Fig. 13: Query Performance vs. Object Cardinality

8.4 Scalability Test on Dimensionality and Object Cardinality

We investigate the impact of dimensionality on the authentication performance using the FoodMarket dataset. We fix the selectivity of candidate objects to 5% of the whole dataset and vary the dimensionality as 1, 3, and 5. For the latter two dimensionalities, we synthesize the non-sensitive attributes based on a uniform distribution and therefore the size of the range query changes accordingly. Figure 12 plots the costs of various aggregate queries. We observe that the performance of *max*, *top-k*, and *FFQ* queries deteriorates severely due to the curse of dimensionality, while the performance of *sum* queries does less. This is because $union(\cdot)$, the most affected operation by dimensionality, is not needed in sum queries.

Finally, Figure 13 shows our scalability test with respect to the cardinality of objects. The results show that the SP CPU time increases monotonically for sum, but is U-shape for other types of queries. This is because the other queries involve not only $sum(\cdot)$ operations but also $union(\cdot)$ operations, whose costs decrease with increasing object cardinality under a fixed number of unique features. On the other hand, the CPU time of the client is independent of total number of features, but is positively related to the multiplicity of the returned multiset result. This also causes a relative high cost in client CPU time for *FFQ* queries, similar to what we have observed in query performance on TPC-H dataset. As for the VO size, it is almost constant as it only depends on the number of OMG-tree nodes that cover the selected candidate objects.

9 Conclusions

In this paper, we have studied the problem of privacy-preserving authentication of aggregate queries over set-valued data. We presented a family of privacy-preserving protocols on multiset operations. Based on that, we proposed PA^2, a framework that supports various privacy-preserving aggregate queries. We also designed the MG-tree to guarantee the correctness of the candidate objects selected by a range query and developed several advanced optimization techniques. Analytical models and empirical results validate the feasibility and robustness of our proposals. In particular, the overhead of authenticating aggregate queries at the client side is only sublinear to the number of candidate objects and is independent of the cardinality of multisets. The evaluation results also reveal that the authentication overhead incurred by the confidentiality preservation requirement is only moderate.

As for future work, we plan to extend the proposed privacy-preserving authentication techniques to more complex aggregate queries such as median and percentile. We also plan to leverage GPGPU technologies to accelerate the computation of accumulative values during query processing at the SP.

Acknowledgement

This work was supported by Research Grants Council (RGC) of Hong Kong under GRF Projects 12244916, 15238116, 12202414, 12200914, CRF Project C1008-16G, and NSFC Grant 61370231.

References

[1] "Google Inc. What is bigquery?" https://cloud.google.com/bigquery/what-is-bigquery, 2016.

[2] "Personal genome project," http://www.personalgenomes.org, 2016.

[3] H. Pang and K.-L. Tan, "Authenticating query results in edge computing," in *Proc. ICDE*, 2004.

[4] F. Li, G. Kollios, and L. Reyzin, "Dynamic authenticated index structures for outsourced databases," in *Proc. SIGMOD*, 2006, pp. 121–132.

[5] Y. Yang, S. Papadopoulos, D. Papadias, and G. Kollios, "Authenticated indexing for outsourced spatial databases." *VLDBJ*, vol. 18, no. 3, pp. 631–648, 2009.

[6] F. Li, M. Hadjieleftheriou, G. Kollios, and L. Reyzin, "Authenticated index structures for aggregation queries," *ACM TISSEC*, vol. 13, no. 32, pp. 1–35, 2010.

[7] S. Papadopoulos, G. Cormode, A. Deligiannakis, and M. Garofalakis, "Lightweight authentication of linear algebraic queries on data streams," in *Proc. SIGMOD*, 2013, pp. 881–892.

[8] Y. Tang, T. Wang, L. Liu, X. Hu, and J. Jang, "Lightweight authentication of freshness in outsourced key-value stores," in *Proc. ACSAC*, 2014.

[9] Q. Chen, H. Hu, and J. Xu, "Authenticated online data integration services," in *Proc. SIGMOD*, 2015, pp. 167–181.

[10] Y. Peng, Z. Fan, B. Choi, J. Xu, and S. S. Bhowmick, "Authenticated subgraph similarity searchin outsourced graph databases," *IEEE TKDE*, vol. 27, pp. 1838–1860, 2015.

[11] R. Agrawal and R. Srikant, "Fast algorithms for mining association rules in large databases," in *Proceedings of VLDB*, 1994, pp. 487–499.

[12] H. Bast and I. Weber, "Type less, find more: Fast autocompletion search with a succinct index," in *Proc. SIGIR*, 2006.

[13] A. Metwally and C. Faloutsos, "V-smart-join: A scalable mapreduce framework for all-pair similarity joins of multisets and vectors," in *Proc. VLDB*, 2012, pp. 704–715.

[14] G. Özsoyoğlu, Z. M. Özsoyoğlu, and V. Matos, "Extending relational algebra and relational calculus with set-valued attributes and aggregate functions," *ACM Transactions on Database Systems (TODS)*, vol. 12, no. 4, pp. 566–592, 1987.

[15] R. Agrawal, T. Imielinski, and A. Swami, "Mining association rules between sets of items in large databases," in *Proceedings of ACM SIGMOD*, 1993.

[16] G. S. Manku and R. Motwani, "Approximate frequency counts over data streams," in *Proceedings of VLDB*, 2002, pp. 346–357.

[17] R. Baden, A. Bender, N. Spring, B. Bhattacharjee, and D. Starin, "Persona: An online social network with user-defined privacy," in *Proc of the ACM SIGCOMM*, 2009, pp. 135–146.

[18] S. Debnath, N. Ganguly, and P. Mitra, "Feature weighting in content based recommendation system using social network analysis," in *Proc of the 17th International Conference on World Wide Web*, 2008, pp. 1041–1042.

[19] G. Yang, Y. Cai, and Z. Hu, "Authentication of function queries," in *Proc. ICDE*, 2016.

[20] R. C. Merkle, "A certified digital signature," in *Proc. Crypto*, 1989, pp. 218–238.

[21] M. L. Yiu, E. Lo, and D. Yung, "Authentication of moving knn queries," in *ICDE*, 2011, pp. 565–576.

[22] H. Pang and K. Mouratidis, "Authenticating the query results of text search engines," in *VLDB*, 2008.

[23] H. Hu, J. Xu, Q. Chen, and Z. Yang, "Authenticating location-based services without compromising location privacy," in *Proc. SIGMOD*, 2012, pp. 301–312. [Online]. Available: http://doi.acm.org/10.1145/2213836.2213871

[24] Q. Chen, H. Hu, and J. Xu, "Authenticating top-k queries in location-based services with confidentiality," in *Proc. VLDB*, 2014, pp. 49–60.

[25] C. Papamanthou, R. Tamassia, and N. Triandopoulos, "Optimal verification of operations on dynamic sets," in *Proc. CRYPTO*, 2011, pp. 91–110.

[26] R. Canetti, O. Paneth, D. Papadopoulos, and N. Triandopoulos, "Verifiable set operations over outsourced databases," in *Proc. PKC*, 2014, pp. 113–130.

[27] D. Papadopoulos, S. Papadopoulos, and N. Triandopoulos, "Taking authenticated range queries to arbitrary dimensions," in *Proc. CCS*, 2014.

[28] B. Dong, R. Liu, and W. Wang, "Integrity verification of outsourced frequent itemset mining with deterministic guarantee," in *Proc. ICDM*, 2013, pp. 1025–1030.

[29] P. Fauzi, H. Lipmaa, and B. Zhang, "Efficient non-interactive zero knowledge arguments for set operations," in *Financial Cryptography and Data Security*, 2014.

[30] E. Ghosh, O. Ohrimenko, D. Papadopoulos, R. Tamassia, and N. Triandopoulos, "Zero-knowledge accumulators and set operations," in *IACR Cryptology ePrint*, 2015.

[31] Y. Zhang, J. Katz, and C. Papamanthou, "An expressive (zero-knowledge) set accumulator," in *EuroS&P*, 2017, pp. 158–173.

[32] D. E. Knuth, *The Art of Computer Programming, Volume 2 (3rd Ed.): Seminumerical Algorithms*. Addison-Wesley Longman Publishing Co., Inc., 1997.

[33] L. Nguyen, "Accumulators from bilinear pairings and applications," in *Proc. CT-RSA*, 2005, pp. 275–292.

[34] C. Papamanthou, R. Tamassia, and N. Triandopoulos, "Authenticated hash tables based on cryptographic accumulators," *Algorithmica*, pp. 1–49, 2015.

[35] D. Boneh, X. Boyen, and H. Shacham, "Short group signatures," in *Proc. CRYPTO*, 2004, pp. 41–55.

[36] N. Bitansky, R. Canetti, A. Chiesa, and E. Tromer, "From extractable collision resistance to succinct non-interactive arguments of knowledge, and back again," in *Proc. the 3rd Innovations in Theoretical Computer Science Conference (ITCS)*, 2012, pp. 326–349.

[37] B.-U. Pagel, H.-W. Six, H. Toben, and P. Widmayer, "Towards an analysis of range query performance in spatial data structures," in *Proc. PODS*, 1993, pp. 214–221.

[38] B.-C. Chen, D. Kifer, K. LeFevre, and A. Machanavajjhala, *Privacy-Preserving Data Publishing*. Now Publishers, 2009.

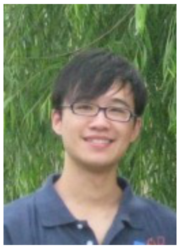
[39] P. Samarati and L. Sweeney, "Protecting privacy when disclosing information: k-anonymity and its enforcement through generalization and suppression," Technical Report SRI-CSL-98-04, SRI Computer Science Laboratory, Tech. Rep., 1998.

[40] N. Li, T. Li, and S. Venkatasubramanian, "t-closeness: Privacy beyond k-anonymity and l-diversity," in *Proc. ICDE*, 2007.

[41] "Foodmarket," http://recsyswiki.com/wiki/Grocery_shopping_datasets, 1998.

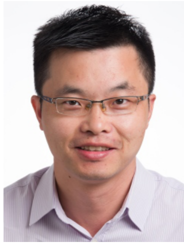
[42] T. P. P. Council, "TPC benchmark H," http://www.tpc.org/tpch/, 2017.

Cheng Xu is a PhD student in the Department of Computer Science, Hong Kong Baptist University and a member of the HKBU Database Research Group (http://www.comp.hkbu.edu.hk/~db). He received his BEng degree from Huazhong University of Science and Technology in 2014. His research interests include information security and privacy-aware computing.

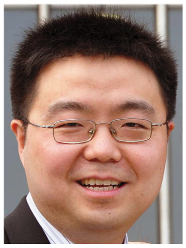


Qian Chen is a PhD student in the Department of Computer Science, Hong Kong Baptist University. He received his BEng degree in Computer Science from East China Normal University in 2011. His research interests include database security and privacy-aware data management.

Haibo Hu is an Assistant Professor in the Department of Electronic and Information Engineering, Hong Kong Polytechnic University. Prior to this, he has held academic positions in HKUST and HKBU since he received his PhD degree from HKUST in 2005. His research interests include information security, privacy-aware computing, wireless data management, and location-based services. He has published over 60 research papers in refereed journals, international conferences, and book chapters. As priciple investigator, he has received over 5 million HK dollars of external research grants from Hong Kong and mainland China. He is the Panel Co-chair of DASFAA 2011, and Program Co-chair of DaMEN 2011, 2013 and CloudDB 2011. He is also the recipient of a number of awards, including ACM-HK Best PhD Paper Award, Microsoft Imagine Cup, and GS1 Internet of Things Award.



Jianliang Xu is a Professor in the Department of Computer Science, Hong Kong Baptist University. He received the BEng degree from Zhejiang University and the PhD degree from Hong Kong University of Science and Technology. He held visiting positions at Pennsylvania State University and Fudan University. His current research interests include data management, database security and privacy, and location-aware computing. He has published more than 150 technical papers in these areas. He has served as a program co-chair/vice chair for a number of major international conferences including IEEE ICDCS 2012, IEEE CPSNA 2015 and WAIM 2016. He is an Associate Editor of IEEE Transactions on Knowledge and Data Engineering (TKDE) and PVLDB 2018.



Xiaojun Hei received the B.Eng. degree in information engineering from the Huazhong University of Science and Technology, Wuhan, China, in 1998, the M.Phil. degree in electrical and electronic engineering from The Hong Kong University of Science and Technology, Hong Kong, in 2000, and the Ph.D. degree from the Department of Electronic and Computer Engineering, The Hong Kong University of Science and Technology in 2008. Since 2008, he has been with the Internet Technology and Engineering Research and Development Center, School of Electronic Information and Communications, Huazhong University of Science and Technology. He is currently an Associate Professor with the School of Electronic Information and Communications, Huazhong University of Science and Technology.

APPENDIX A PROOF OF THEOREM 1

We start by proving the security of the privacy-preserving authentication protocols on multiset operations. Note that the proofs of $sub(\cdot,\cdot)$ and $empty(\cdot)$ are similar to those on set operations in [25] and are hence omitted.

A.1 Security of $sum(\cdot)$ Operation

Lemma 2 (Security of $sum(\cdot)$ ***operation).*** Under the bilinear q-strong Diffie-Hellman assumption, if the accumulative value returned by $sum(\{X_1,\cdots,X_n\})$ passes the client's verification, then the probability of $sum(\{X_1,\cdots,X_n\}) \neq acc(\uplus_{i=1}^{n} X_i)$ is negligible for any PPT adversary.

PROOF. First, we prove that this lemma holds for $n=2$ by contradiction. Support there is a PPT algorithm that computs such a multiset $S=\{y_1,\ldots,y_\ell\}$, whereas $y_j \notin X_1 \uplus X_2$ for some $1 \le j \le \ell$. This means that

$$e(acc(X_1), acc(X_2)) = e(g^{\prod_{y\in S}(y+s)}, g) \Rightarrow$$
$$e(g,g)^{\prod_{x\in X_1 \uplus X_2}(x+s)} = e(g,g)^{\prod_{y\in S}(y+s)}$$

Note that $(y_j+s) \nmid \prod_{x\in X_1\uplus X_2}(x+s)$. Therefore, there exists a polynomial $Q(s)$ (computable in polynomial time) of degree $n-1$ and constant $\lambda \neq 0$, such that $\prod_{x\in X_1\uplus X_2}(x+s) = Q(s)(y_j+s)+\lambda$. Thus, we have

$$e(g,g)^{(y_j+s)\prod_{1\le i\neq j\le \ell}(y_i+s)} = e(g,g)^{Q(s)(y_j+s)+\lambda} \Rightarrow$$
$$e(g,g)^{1/(y_j+s)} = \left[e(g,g)^{\prod_{1\le i\neq j\le\ell}(y_i+s)} e(g,g)^{-Q(s)}\right]^{\lambda^{-1}}.$$

Thus, this algorithm can break the bilinear q-strong Diffie-Hellman assumption. This proves that our lemma holds for $n=2$.

Supposing our lemma is true for any $n=k$, where $k\ge 2$, in the following we prove it is also true for $n=k+1$. Let ε_k denote the event that Adv returns incorrect accumulative value $sum(\uplus_{i=1}^{k} X_i)$ and passes the client's verification. By law of probability, we have:

$$\begin{aligned} Pr[\varepsilon_{k+1}] &= Pr[\varepsilon_k]Pr[\varepsilon_{k+1}|\varepsilon_k] + Pr[\varepsilon'_k]Pr[\varepsilon_{k+1}|\varepsilon'_k] \\ &\le Pr[\varepsilon_k] + Pr[\varepsilon_{k+1}|\varepsilon'_k] \\ &= Pr[\varepsilon_k] + Pr[\varepsilon_2], \end{aligned}$$

where ε' denotes the complement of an event. Finally, we have:

$$Pr[\varepsilon_n] \le (n-1)Pr[\varepsilon_2]$$

Since $Pr[\varepsilon_2]$ is negligible according to our proof for $n=2$, we can conclude that $Pr[\varepsilon_n]$ is negligible as $p \gg n$ in a cyclic multiplicative group $\mathbb{G}$. Hence, this lemma is proved. □

A.2 Security of $union(\cdot)$ Operation

Lemma 3 (Security of $union(\cdot)$ ***operation).*** Under the bilinear q-strong Diffie-Hellman assumption, if the accumulative value returned by $union(\{X_1,\cdots,X_n\})$ passes the client's verification, then the probability of $union(\{X_1,\cdots,X_n\}) \neq acc(\cup_{i=1}^{n} X_i)$ is negligible for any PPT adversary.

PROOF. Let $U=\cup_{i=1}^{n} X_i$. The $union(\cdot)$ protocol consists of three modules (i) $sub(\widehat{X}_1, U)$, $sub(\widehat{X}_2, U)$, $\cdots$, $sub(\widehat{X}_n, U)$; (ii) $empty(\{U-\widehat{X}_1, U-\widehat{X}_2,\cdots,U-\widehat{X}_n\})$; and (iii) SP sends the ECRH hash value $h_e(U)^*$ for U. Due to the composition property and the security of (i)(ii), we only need to prove the security of (iii), that is, $acc(U)^*$ cannot be forged if $h_e(U)^*$ passes the client's verification. Denote $acc^{-1}(acc(U)^*)$ as the set who has accumulator value $acc(U)^*$.

First, we prove $acc^{-1}(acc(U)^*) \subseteq U$, i.e., the returned value $acc(U)^* = g^{P(U)^*}{=}g^{P(U)\cdot Q_U(s)}$, where $Q_U(s)$ is some polynomial. We prove this by contradiction. We assume there exists an element $x_j \in \widehat{X}_i$ such that $(x_j+s) \nmid P(U)^*$, i.e., $P(U)^* = (x_i+s)\cdot Q_i(s)+\lambda_i$, where $Q_i(s)$ is some polynomial and λ_i is a constant value. During the execution of protocol $sub(\widehat{X}_i, U)$, by expanding $e(acc(\widehat{X}_i), W_i^*) = e(acc(U)^*, g)$, the adversary can get:

$$e(g^{\prod_{k=1}^{|\widehat{X}_i|}(x_k+s)}, W_i^*) = e(g,g)^{(x_j+s)\cdot Q_i(s)+\lambda_i}.$$

By dividing (x_j+s) on the exponents of both sides, the adversary can get:

$$e(g,W_i^*)^{\prod_{k=1}^{j-1}(x_k+s)\cdot\prod_{k=j+1}^{|\widehat{X}_i|}(x_k+s)} = e(g,g)^{Q_i(s)+\lambda_i/(x_j+s)}.$$

Finally, the adversary can get:

$$\begin{aligned} &e(g,g)^{1/(x_j+s)} \\ &{=}(e(g,W_i^*)^{\prod_{k=1}^{j-1}(x_k+s)\cdot\prod_{k=j+1}^{|\widehat{X}_i|}(x_k+s)}\cdot e(g,g)^{-Q_i(s)})^{1/\lambda_i}, \end{aligned}$$

which violates the bilinear q-strong Diffie-Hellman assumption mention in Section 2. Hence, by contradiction, $P(U)^*$ must be in the form of $P(U)\cdot Q_U(s)$.

Next, we prove $U \subseteq acc^{-1}(acc(U)^*)$, i.e., $Q_U(s)$ can only be a constant value. We prove this by contradiction. For each set $\widehat{X}_i$, $acc(U-\widehat{X}_i)^* = g^{P(U-\widehat{X}_i)\cdot Q_U(s)}$. During the execution of $empty(\cdot)$, by expanding $\prod_{i=1}^{n} e(acc(U-\widehat{X}_i)^*, F_i^*) = e(g,g)$, the adversary can get:

$$\prod_{i=1}^{n} e(g^{P(U-\widehat{X}_i)}, g^{Q_i(s)}) = e(g,g),$$

and then

$$e(g,g)^{Q_U(s)(\prod_{i=1}^{n} P(U-\widehat{X}_i)\cdot Q_i(s))} = e(g,g).$$

Finally, the adversary can get:

$$e(g,g)^{1/Q_U(s)} = e(g,g)^{P(U-\widehat{X}_i\cdot Q_i(s))}.$$

Since $Q_U(s)$ is a polynomial whose order is larger than 1, the adversary violates the bilinear q-strong Diffie-Hellman assumption. Thus, by contradiction, $Q_U(s)$ must be a constant value.

By the above two arguments, we prove that the accumulative value $acc(U)^*$ cannot be forged if it passes the client's verification. □

A.3 Security of $times(\cdot,\cdot)$ Operation

Lemma 4 (Security of $times(\cdot,\cdot)$ ***operation).*** Under the bilinear q-strong Diffie-Hellman assumption, if the accumulative value returned by $times(X,t)$ passes the client's verification, then the probability of $times(X,t) \neq acc(\uplus_{i=1}^{t} X)$ is negligible for any PPT adversary.

PROOF. The $times(\cdot,\cdot)$ operation relies only on the $sum(\cdot)$ operation, and therefore the proof is similar to that of Lemma 2. □

A.4 Security of PA^2 Algorithms

Following the above lemmas, we now show that the query results returned by the PA^2 algorithms cannot be forged.

Theorem 1. The PA^2 authentication algorithms for aggregate queries guarantee the correctness and soundness of the query results under the bilinear q-strong Diffie-Hellman assumption.

PROOF. According to the lemmas on the security of operations $sub(\cdot,\cdot)$, $empty(\cdot)$, $sum(\cdot)$, $union(\cdot)$, $times(\cdot,\cdot)$, the theorem is trivially true due to the composition property. □